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ABSTRACT

These instructional objectives have been selected from materials submitted to the Curriculum Laboratory of the Graduate School of Education at UCLA. Arranged by major course goals, these objectives are offered simply as samples that may be used where they correspond to the skills, abilities, and attitudes instructors want their students to acquire. These objectives may also serve as models for assisting instructors to translate other instructional units into specific measurable terms. For other objectives in related courses see: ED 033 683 (College Algebra); ED 033 687 (Calculus and Analytic Geometry); ED 033 698 (Geometry); JC 710 120 (College Mathematics); and JC 710 129 (Intermediate Algebra). (MB)

Instructional Objectives for a Junior College Course in Introduction to Mathematical Thinking

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INTRODUCTION TO MATHEMATICAL THINKING

Unit I

Mathematical Systems

Mathematical systems are based on undefined terms, defined terms, axioms, and theorems. The concepts of abstraction, logic, structure of mathematical statements, and rigor of treatment are also important for understanding fundamentals.

Objectives

<u>Goal</u>: The student will understand some of terms and concepts basic to any mathematical system.

<u>Objectives</u>:

(1) The student will define four of the following six terms: axioms, undefined terms, theorems, consistent, independent, rigor. 75%

(2) The student will list the undefined terms, definitions, and axioms of the abstract system given on page 6 of the text and will state a theorem for this system not given in the text on an outside assignment. Any reference may be used. 100%

Unit 2

Introduction to Logic

Mathematics has a precise language. In order to read and understand this language, we must study its structure. To do this we consider various statements as to their meaning in symbolic form.

Objectives

Goal: The student will know the various types of mathematical statements and understand their structure and their truth values.

Objectives:

(1) When given two statements the student will determine whether one condition is necessary for another condition, sufficient for another condition, or both necessary and sufficient. The statement will then be rephrased by the student into statements of the form.
a. If . . . , then . . . or
b. . . . if and only if . . . 100%

Example: Whenever it rains it pours.
"It rains" sufficient for "It pours."
"It pours" necessary for "It rains."

If it rains then it pours.

(2) When given three statements the student will write the negation of the statements. 100%

Example:

Statement: All men are rogues.
Negation: Some men are not rogues.

(3) The student will disprove a statement by giving a counter example. 100%

(4) The student will give an example of a conjunctive statement, an exclusive disjunctive statement and an inclusive disjunctive statement. 100%

(5) When given a statement in symbolic form, the student will calculate a truth table for the statement. 100%

Example: $(A \wedge \sim B) \rightarrow (\sim B \vee A)$

Solution:

A	B	~B	(A ∧ ~B)	→	(~B V A)
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	F
F	F	T	F	T	T

(The final truth table column is circled.)

(6) Given two statements, the student will determine whether or not they are logically equivalent by the use of truth tables. 100%

(7) The student will define four of the following terms: sufficient condition, necessary condition, negation, counterexample, conjunction, exclusive disjunction, inclusive disjunction, tantology, logical equivalence. 75%

Unit 3

Proofs and Arguments

All statements made in mathematics other than our basic axioms and definitions are derived by proof using logical arguments. Therefore, to understand mathematical thinking, it is necessary to be able to analyze proofs and the arguments which compose them.

Objectives

Goals: The student will understand what constitutes a valid proof as well as some of the formal logical arguments acceptable in proofs.

Objectives:

(1)* Given a statement the student will form the converse and the contrapositive of the statement. 100%

(2)† Given an argument in symbolic form the student will make up a truth table for each premise and for the conclusion and then verify the validity of the argument. 100%

(3) Given four arguments the student will decide if each is valid or invalid by referring to one of the following classical forms. 75%

- (a) Valid--Modus Ponens
  - Modus Tollens
  - Hypothetical Syllogism
  - Disjunctive Syllogism
  - Constructive Dilemma
  - Destructive Dilemma
- (b) Invalid--Affirming the Consequent
  - Denying the Antecedent

(4) Given two proofs by contradiction the student will analyze them and determine in each case whether the argument is valid. 100%

(5) The student will define three of the following terms: converse, contrapositive, premises, antecedent, consequent. 100 %

Extra Objectives

Objectives:

E_1 The student will work all the exercises on pages 60-61 of the text on an outside class assignment due one week following the conclusion of this unit. Any reference may be used. (7 out of 9)

E_2 The student will make up two arguments which end with the conclusion "the Butler did it." Make one of the arguments valid and one invalid. These are due no more than one week from the completion of this unit. Any reference allowed.

Unit 4

Sets, Set Operations

The concept of set underlies and forms the basis of all mathematical systems. This unit reviews all the basic properties of naive set theory.

Objectives

Goal: The student will understand set concepts and set operations.

Objectives:

(1) Given three collections of objects, the student will decide which of these collections form sets. 100%

(2) Given three sets the student will compute four problems of intersection, union, and difference of these sets and make a Venn diagram for the sets. 75%

(3) The student will define four of the following terms: set, element, subset, proper subset, empty set, intersection, union, difference of two sets. 75%

Extra Objectives

Objective: E_3 The student will draw a Venn Diagram for the following problem and answer the questions. This objective is due within one week following the end of the unit. 100%

Problem: Suppose that in a survey exploring the reading habits of students it is found that:

60% read magazine A	20% read magazines B and C
50% read magazine B	30% read magazines A and C
50% read magazine C	10% read all three magazines
30% read magazines A and B	

a. What per cent read exactly two magazines?

b. What per cent do not read any of the magazines?

Unit 5

Sets and Logic

The study of symbolic logic is closely related to the study of sets. We can use set theory to diagram logical statements. This allows us to better visualize proofs by using Venn diagrams.

Objectives

Goal: The student will be able to convert logical statements to set theory statements and analyze arguments using diagrams called Venn diagrams.

Objectives:

(1) Given three sets the student will diagram these sets and label five new sets made from unions, intersections, and differences of these original sets. 80%

(2) Given three arguments the student will convert these to set theory statements and draw Venn diagrams of them. The student will then analyze the diagrams and decide which are valid and which are invalid. 80%

Unit 6

Counting Numbers

The notion of set gives rise to the concept of counting or matching two sets. This concept gives rise to the concept of the cardinal number. Some properties of addition of cardinal numbers are proved in this unit.

Objectives

Goal: The student will understand the natural development of cardinal numbers and the operation of addition from the concepts of sets and counting.

Objectives:

(1) The student will select two pairs of two sets each in such a way that the first pair has the same number of elements and the second pair will not have the same number of elements. In both cases the student will make up a pairing to illustrate that the sets do or do not have the same number of elements. 100%

(2) The student will answer ten true-false questions about countability. 80%

(3) The student will answer five true-false questions about cardinality. 80%

Extra Objectives

Objectives:

E_4 The student will work problem 4, page 122 of the text, on an outside assignment which will be due one week after the end of the unit. If the definition in this problem is not good, make up one that will work in all cases. 100%

E_5 The student will develop a mathematical system for Cardinal numbers stating all definitions and propisitions needed from set theory and the definitions of Cardinality and addition; and will prove Propositions 10, 11, 12, and 13 on pages 119-121 of the text. 100%

Unit 7

The Cartesian Product--Functions

The Cartesian Product is a different kind of operation on two sets. It is the basis of the study of functions and also gives us a method for defining the operation of multiplication of cardinal numbers.

Objectives

Goal: The student will understand the concepts and terminology of mathematical functions and will gain a firm foundation in what is meant by multiplication of cardinal numbers.

Objectives:

(1) Given two sets the student will compute a cross product of the two sets. 100%

(2) Given two sets the student will list all the functions from one set into another set.

(3) Given a function f specified by a rule from one set S to another set T, the student will make a list of all ordered pairs of the function and specify the image of S under f.

(4) Given three sets S, T, and W, the student will decide which sets may be used to form onto functions from one set to another and which may be used to from one-one functions. Next the student will form a composite function, i.e. a function f from S into T and a function g from T into W such that $g \circ f$ is a function from S to W.

(5) The student will define five of the following terms: Cartesian product, first coordinate, second coordinate, function, into, onto, one-one, image, inverse, and composition of two functions. 80%

Extra Objectives

Objective: E_6 The student will continue the development of cardinal numbers by stating the definition of the Cartesian product of two sets and the definition of multiplication and prove the propositions 2, 3, 4, 5, and 6 on pages 127-130 of the text.

Unit 8

Relations

The notion of relation is similar and equal in importance in mathematics to that of function. From the concept of relation we develop the concept of equivalence relations and integers. Relations help us understand the ordering properties of numbers.

Objectives

Goals:

(1) The student will understand the concept of a relation and especially equivalence relations and partial orderings.

(2) The student will gain some insight into the development of the integers from the cardinal numbers.

Objectives:

(1) Given three relations the student will decide whether or not it is an equivalence relation and if not, which properties of an equivalence relation are missing. 66%

(2) Given integers in their ordered pair form, the student will (a) work two problems of addition, (b) work two problems of mutiplication, and (c) decide if any of the given integers belong to the same equivalence class. 100%

(3) Given a relation R on a set S the student will show that R is totally ordered, partially ordered, or neither. 100%

(4) The student will define five of the following terms: relation, equivalence class, partial ordering, total ordering, equivalence relation, upper bounds, least upper bound, lower bound, greatest lower bound, R-related. 80%

Extra Objectives

Objective: E_7 The student will continue the development of numbers from cardinals to integers by assuming definition 8.4 and by proving propositions 3, 4, 5, and 6. 100%

Unit 9

Total Ordering

The integers form a totally ordered set as well as a partially ordered one. The properties of a totally ordered set are very important in mathematics. We shall examine some of these properties, particularly mathematical induction.

Objectives

Goals:

(1) The student will gain some insight into the properties of a totally ordered set.

(2) The student will understand and be able to use the proof by mathematical induction on simple problems.

Objectives:

(1) Given a simple statement concerning a progression the student will prove that the statement is true for all elements of the progression. 100%

Example: Show that if n is any odd integer, then:

$$1 + 3 + 5 + . . . + n = n^2$$

(2) The student will define four of the following terms: sucessor, predecessor, discrete set, first element, last element, progression, regression. 75%

Extra Objectives

Objective: E_8 The student will give an example of a discrete set and a set which is not discrete. He will then explain why each set is or is not discrete. 100%

Unit 10

Probability

The concept of probability is used in mathematics in the area of statistics. It is used in the course as a further demonstration of a mathematical system.

Objectives

Goal: The student will understand the basic rules of probability and be able to use them.

Objectives:

(1) Given an experiment with two possible outcomes, A and B, the probabilities of which are given, the student will compute:

a.	$P(A,B)$	e.	$P(A) + P(B)$
b.	$P(\bar{A},B)$	f.	$P(A + B)$
c.	$P(A,\bar{B})$	g.	$P(A \mid B)$
d.	$P(\bar{A},\bar{B})$		

100%

(2) Given two problems on counting possible outcomes, the student will decide if the problem is one of permutations or combinations and work each problem. 100%

(3) The student will define three of the following terms: experiment, trial, event, relative frequency, combinations, and permutations. 100%

Extra Objectives

Objective: E_9 The student will calculate the probability of of four people selected being born in the same month. 100%

Unit 11

An Elementary Geometry

Geometry has come a long way from Euclid. Today there are many geometries which exist as mathematical systems. We shall consider a simple one which will further show how basic mathematical thinking is used. This will be our only truly Axiomatic study.

Objectives

Goals:

(1) The student will learn that there is more to geometry than the Euclidean variety.

(2) The student will understand the projective planes and the affine plane.

Objectives:

(1) The student will list the five axioms of the projective plane. 100%

(2) The student will list the five axioms of the affine plane.

(3) The student will note the difference in projective geometry and affine geometry and give at least one theorem for each that is not true in the other on an outside assignment due the class period following the one in which it is assigned.

Unit 12

Conclusion

The question "Where do we go from here?" is answered in part in this unit. No student can consider this the last mathematics he will learn. As long as one lives, mathematical concepts will still be encountered. This course should help the student understand some of these concepts more deeply.

Objectives

Goal: The student will have changed his thinking about mathematics to one which is more realistic and laced with basic understanding.

Objective: The student will write a two-page critique of his learning in this course. This critique will deal with how this course has changed your basic concept of mathematics and mathematical thinking. (Unless there has been some change, this course has been a waste of time.)